PORTFOLIO DRAWDOWN OPTIMIZATION WITH GENERATIVE MACHINE LEARNING

RFinance
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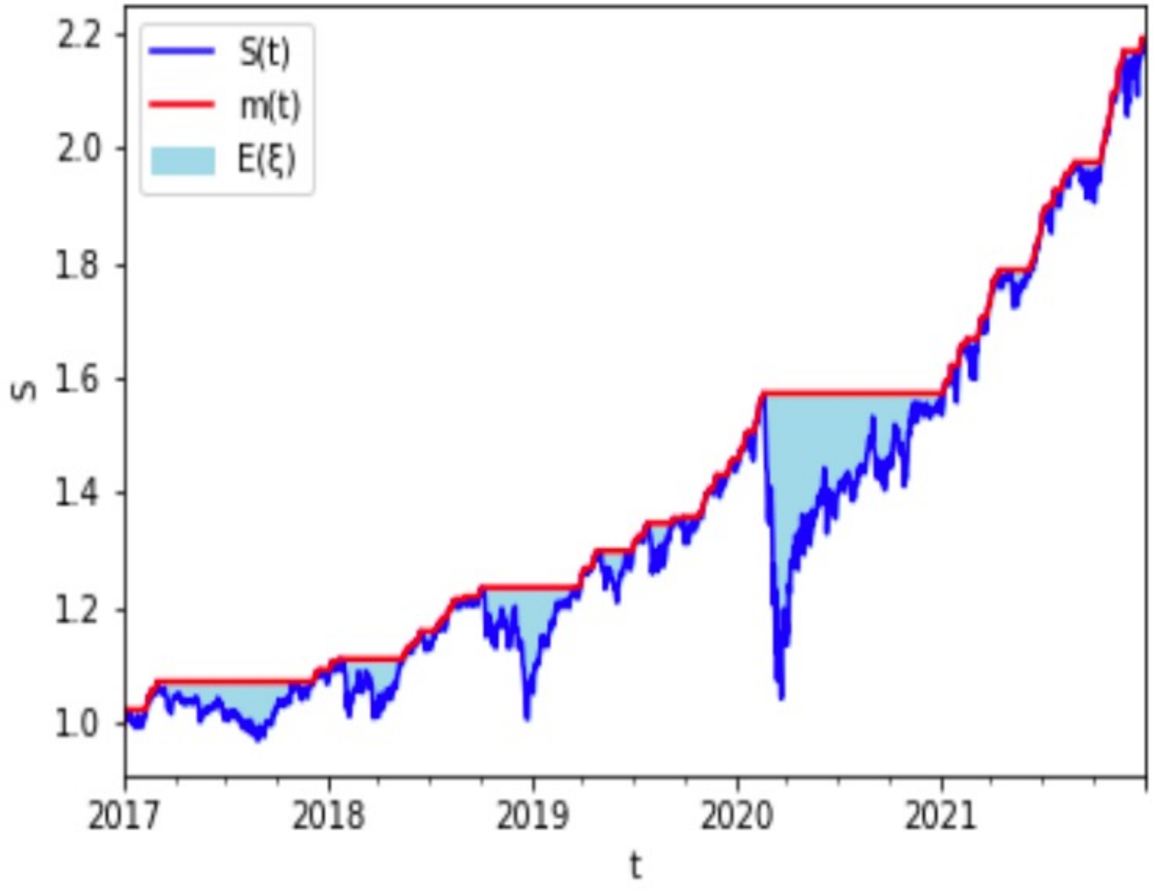
PORTFOLIO DRAWDOWN OPTIMIZATION

- Minimize *expected* peak-to-valley loss

- Minimize frequency, size and duration of losses

- Dynamic generalization of a deviation measure on the path space (Chekhlov et al. 2005)
  $\rightarrow$ path dependence

$$
\begin{aligned}
\min_{\mathbf{w}} \quad & \mathbb{E}(\xi(\mathbf{w})) \\
\text{s.t.} \quad & \xi_{\mathbf{t}} = \mathbf{m}_{\mathbf{t}} - \mathbf{w}\mathbf{S}_t \\
& \mathbf{m}_t \geq \mathbf{m}_{t-1} \\
& \mathbf{m}_t \geq \mathbf{w}\mathbf{S}_t \\
& \mathbf{w}\mathbf{I}^N = 1 \\
& \mathbf{w} > 0
\end{aligned}
$$



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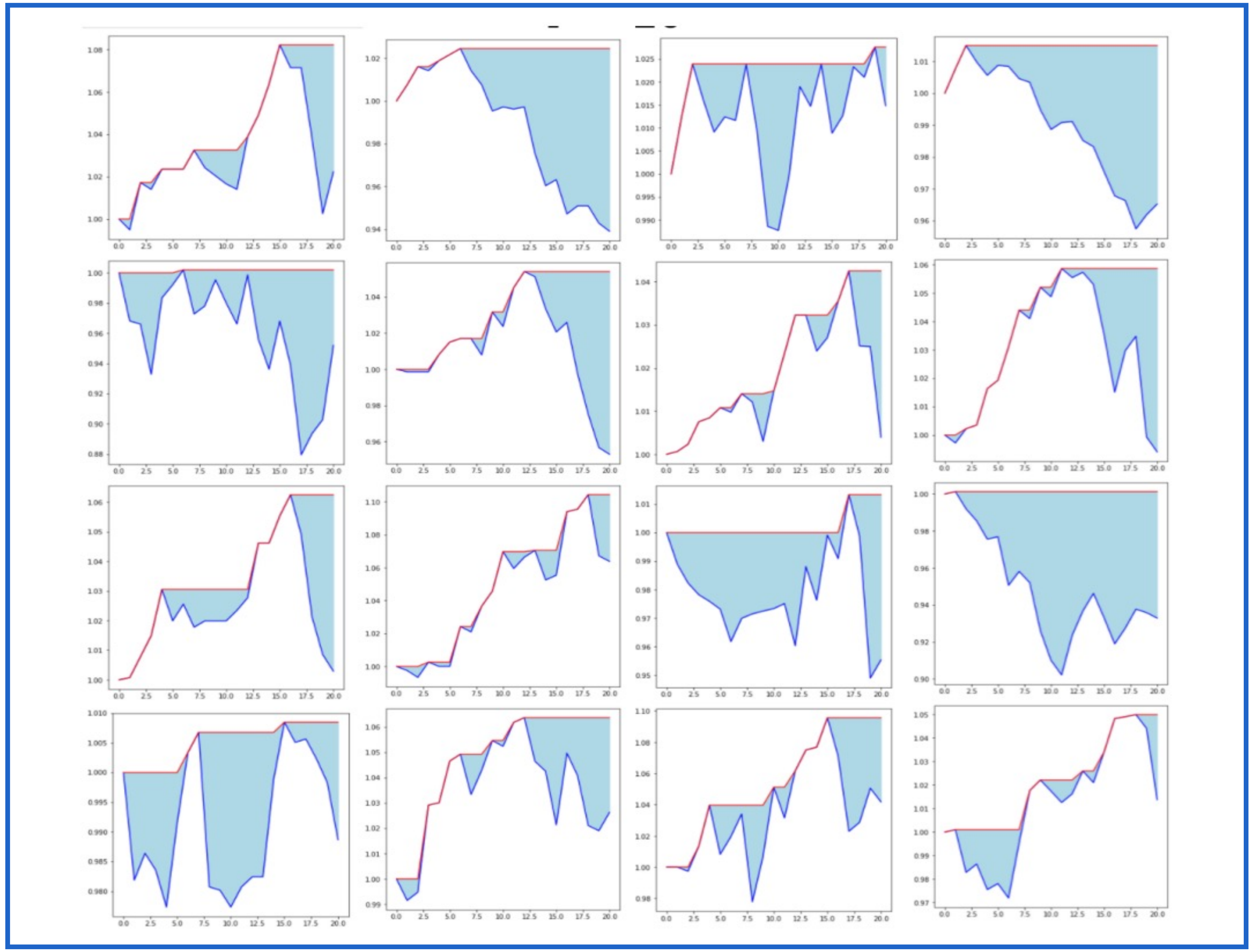
HISTORICAL SIMULATION VS GENERATIVE ML

- Fundamental question: how should we form expectations about the future while looking at the past

- Noise vs. signal

- Unobserved states

- So-called ergodicity assumption



MARKET GENERATOR

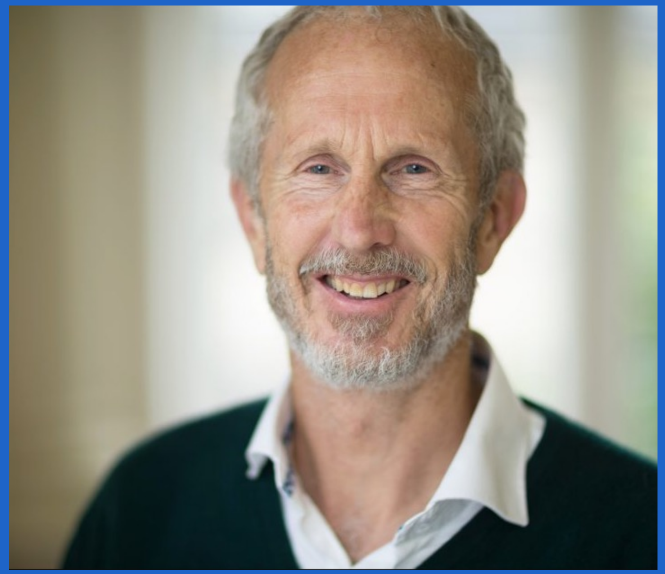
- **Market Generator** =

generative models with the specificity of modelling financial markets

(such as spot asset prices, option prices and volatilities, or order streams in limit order books)

Paper	Year	Architecture	Application
Henry-Labordere [29]	2019	GAN	Option prices
Wiese et al. [30]	2019	GAN	Hedging strategies
Cuchiero et al. [31]	2020	GAN	Volatility models
Ni et al. [32]	2020	GAN	Spot prices
Wiese et al. [33]	2020	GAN	Spot prices
Li et al. [34]	2020	GAN	Order book simulation
Storchan et al. [35]	2020	GAN	Spot prices
Benedetti [36]	2020	GAN	Yield models
Xu et al. [37]	2020	GAN	Spot prices
Pardo and López [38]	2020	GAN	Spot prices
Buehler et al. [39]	2021	GAN	Hedging strategies
Ni et al. [40]	2021	GAN	Spot prices
Pfenninger et al. [41]	2021	GAN	Spot prices
Rosolia and Osterrieder [42]	2021	GAN	Spot prices
Koshiyama et al. [43]	2021	GAN	Spot prices
van Rhijn et al. [44]	2021	GAN	Spot prices
Marti et al. [45]	2021	GAN	Correlation matrices
Coyle et al. [46]	2021	GAN	Spot prices
Wiese et al. [47]	2021	NF	Spot and Option prices
Kondratyev and Schwarz [48]	2019	RBM	Spot prices
Lezmi et al. [49]	2020	RBM / GAN	Spot prices
Wang [50]	2021	RBM / VAE	Spot prices
Buehler et al. [51]	2020	VAE	Spot prices
Fung [52]	2021	VAE	Option prices
Frandsen [53]	2021	VAE	Hedging strategies
Bergeron et al. [54]	2021	VAE	Volatility models
Ning et al. [55]	2021	VAE	Volatility models

Table 1: Overview of the market generator literature



“ SCENARIO-BASED SCIENCE IS MAYBE THE BEST WE CAN DO WHEN DEALING WITH COMPLEX SYSTEMS. ”

DOYNE FARMER

GENERATIVE MACHINE LEARNING

- Approximate data-generating process using flexible (neural) mapping $f_\theta(Z)$

- $f_\theta(Z)$ transports some source distribution of randomness Z (e.g. Gaussian) into data X' that is statistically indistinguishable from the original sample X, according to some metric $\mathcal{L}$.

- E.g. in the distributional sense $\mathrm{P}(X) \approx \mathrm{P}_\theta(X')$, according to some distance $\mathcal{L} = sup_{f \in K} \left|\left|\int f(X)\,\mathrm{P}_\theta(X) d(X) - \int f(X)\,\mathrm{P}(X) d(X)\right|\right|$, e.g. Wasserstein distance, total variation (Kullback-Leibler upper bound by Pinsker's inequality), etc.

- Common neural architectures: **GAN**, **VAE**, **RBM**

MACHINE LEARNING ON PATHS

Signature transform :

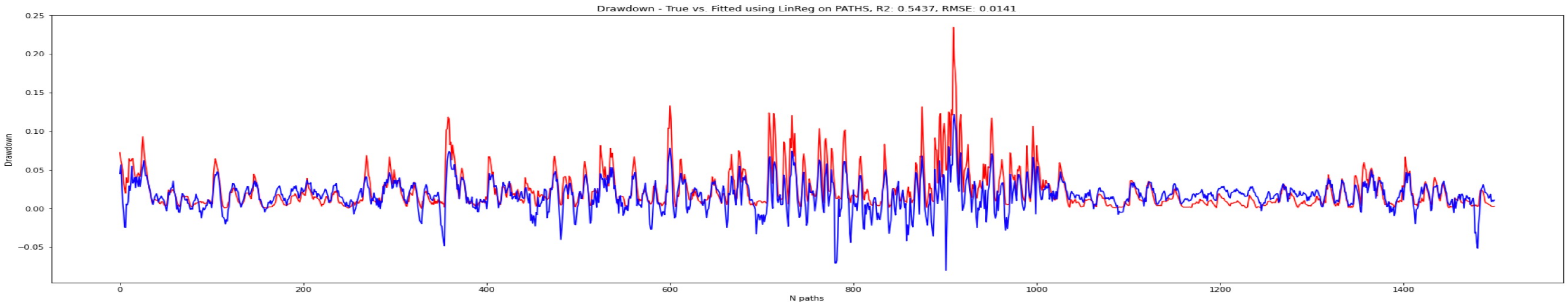
The collection of all iterated integrals of a path that serves as a graded summary of a path describing global and increasingly local properties.

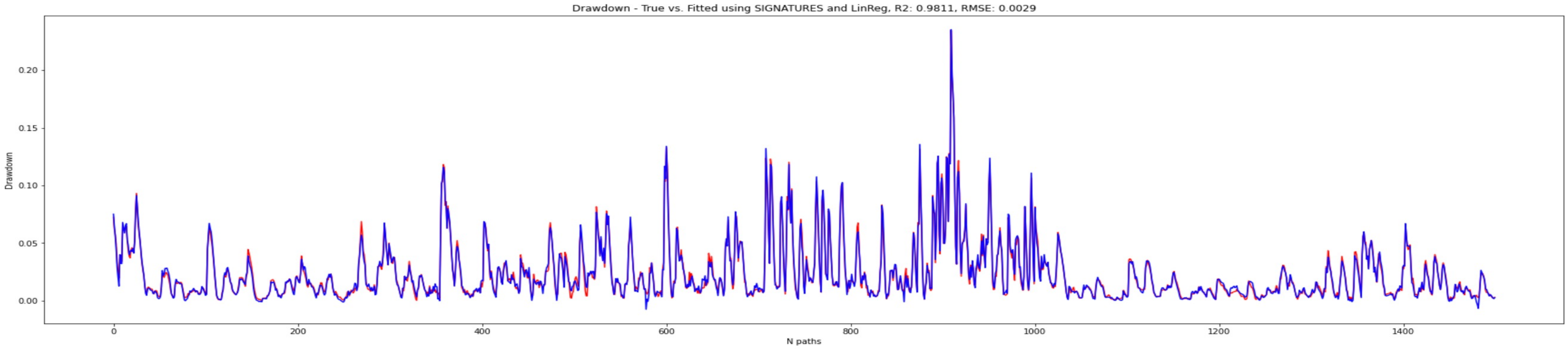
Definition (by Lyons 2014) : $\Phi_M = (1, \Phi^1, \Phi^2, \dots, \Phi^M)$

where $\Phi^n = \iiint_{0<u_1<\dots<u_k<T} d\gamma_{u_1} \otimes \dots \otimes d\gamma_{u_k} \in (\mathbb{R}^d)^{\otimes n}$

Universality (by Levin and Lyons, 2013) **:** $sup_{\gamma \in K} |f(\gamma) - \langle L, \Phi_M(\gamma) \rangle| < \varepsilon$

SIGNATURE UNIVERSALITY AND DRAWDOWN





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MACHINE LEARNING ON PATHS: A SIMPLE MARKET GENERATOR

Algorithm 1 VAE market generator given important path feature Ξ

Input Historical price paths $S : [0, T] \to \mathbb{R}^N$, hyperparameters listed above + signature truncation level M and feature weight α.

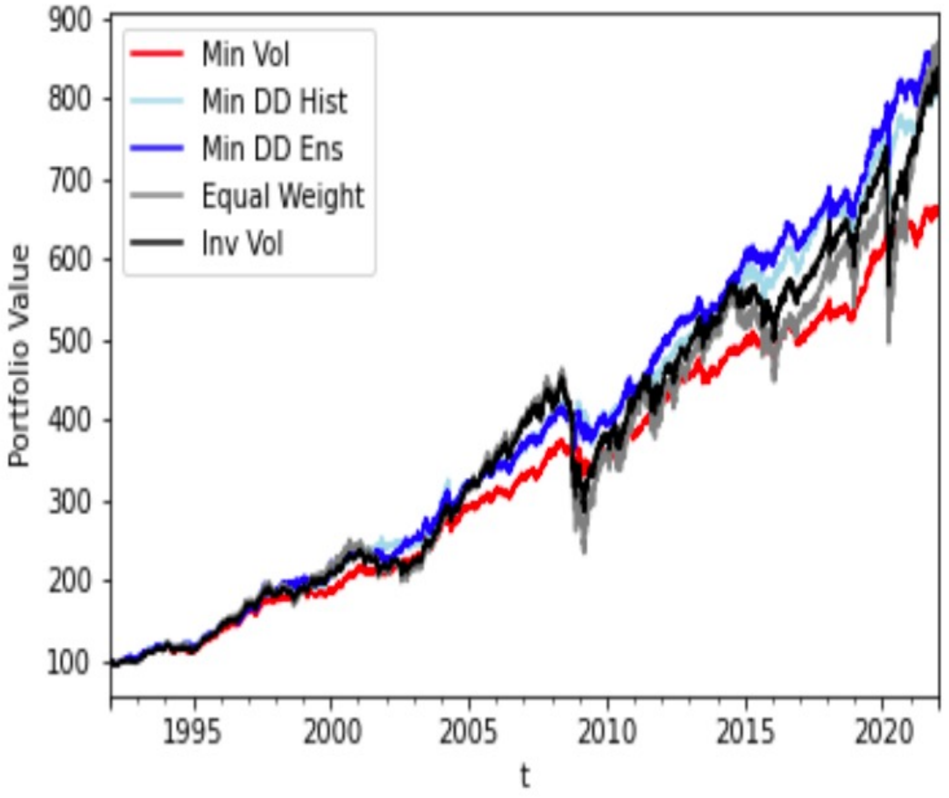
Output Trained VAE Market Generator g_θ

procedure Train
    Divide historical sample into batches $\mathcal{B}$ of length τ, calculate the signatures of these paths truncated at level M, $\Phi_M^{\mathcal{B}}$, calculate the drawdowns Ξ of these paths $\Xi(S^{\mathcal{B}}) = \int_0^\tau (max_{t_i<t}(S_{t_i}^{\mathcal{B}}) - S^{\mathcal{B}})dt$ denoted $\hat{\Xi}(S_b)$
    $\hat{L} \leftarrow LinearRegression(\hat{\Xi}(S^{\mathcal{B}}), \Phi_M^{\mathcal{B}})$
    Initialize the parameters θ of the VAE.
    for $i : \{1, \ldots, N\}$ **do**:
        Sample a batch $\mathcal{B}$ and pass it through the encoder g_θ and decoder network g_θ^{-1} .
        Calculate drawdown $\Xi(S')$ of the output sample S' using the differentiable signature approximation: $\langle \hat{L}, \Phi_M(S') \rangle$
        Define the reconstruction loss term as the weighted average of RMSE error and drawdown loss: $\mathcal{L}_{\mathcal{R}} = \mathbb{E}_{\mathcal{B}}||S - S'||^2 + \alpha \mathbb{E}_{\mathcal{B}}||\langle \hat{L}, \Phi_M(S) \rangle - \langle \hat{L}, \Phi_M(S') \rangle||^2$.
        $\mathcal{L} = \mathcal{L}_L + \mathcal{L}_R$
        $\theta \leftarrow \theta - l\frac{d\mathcal{L}(\theta)}{d\theta}$

PRELIMINARY RESULTS

- 30y backtest on 4 broad asset class indices: US Equity (S&P 500 index), US Bonds (Treasury index), Real Estate (NAREIT index), and Commodities (GSCI).
- Jan 1992 – Jan 2022, monthly rebalanced
- Benchmarks: min vol, inverse vol, equal weight

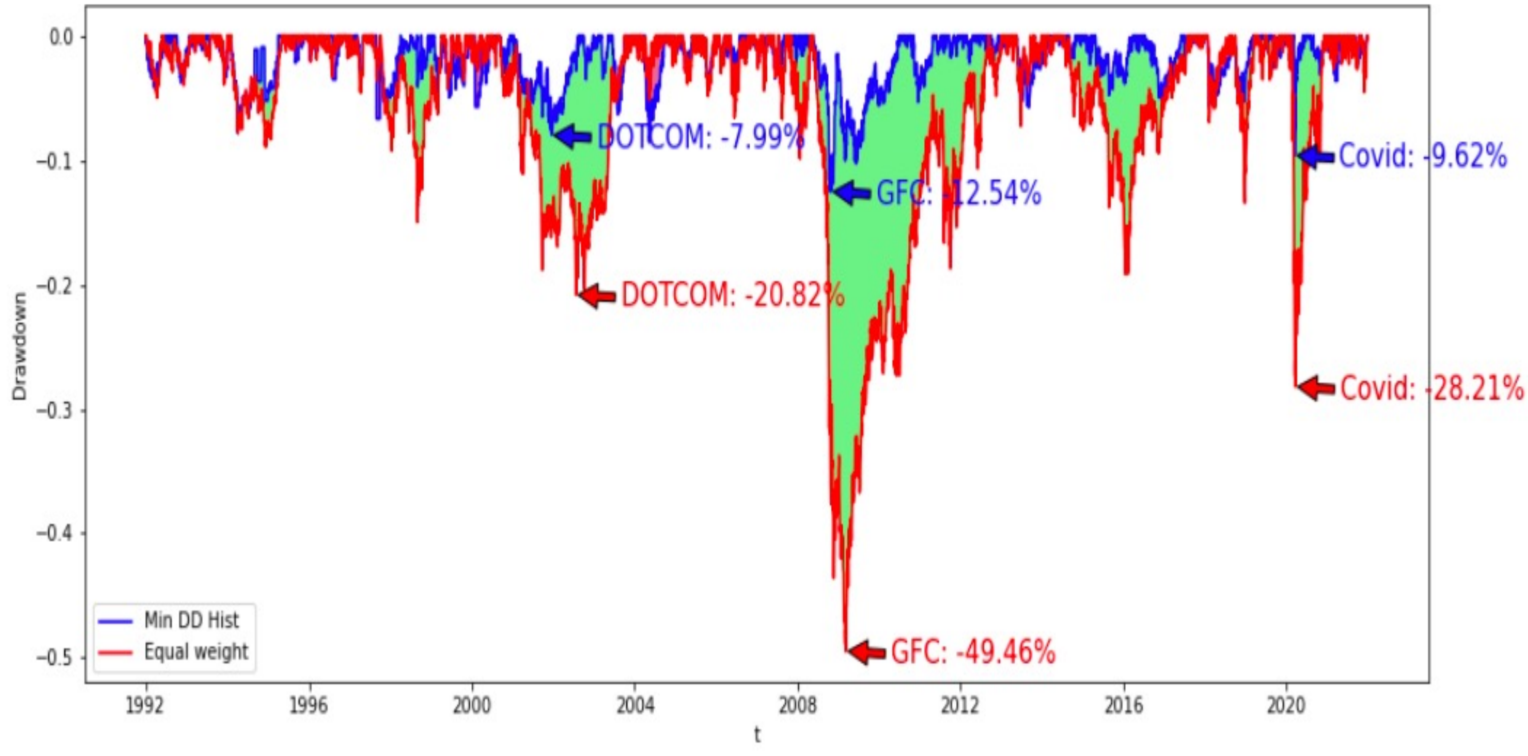
Value



Summary

	Return (%)	Vol (%)	ADD (%)	Sharp
Min Vol	6.187	**4.822**	-2.126	1.283
Min DD Hist	6.967	5.905	-1.911	1.179
Min DD Ens	7.069	5.488	**-1.827**	**1.288**
Equal Weight	**7.427**	10.22	-5.806	0.726
Inv Vol	7.087	7.736	-3.793	0.916

Under-water curve



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